Contributions to the Ontological Foundation of Knowledge Modelling

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Abstract. In the present paper we make some contributions to the emerging field of so-called ontological engineering. In detail, we discuss the principles underlying knowledge acquisition from an axiomatic-deductive point of view. Then we turn to the notion of an ontology and the connection between ontology and knowledge systems. An ontology comprises ontologically basic relations such as membership, part-whole and inherence. The ontological basic entities are divided into Urelemente and sets, the Urelemente are in turn either individuals or universals. Foremost among the individuals are substances, moments and situations. We outline a knowledge modelling language called *GOL*, for (*General Ontological Language*), and we compare it to other languages of this sort.

1 Introduction

In recent years AI research has been developing notions of ontology that play a significant role in knowledge representation and reasoning. Ontologies are generally held to provide definitions for the terms used to represent knowledge. Any sentence, that is not a tautology and that is true of the conceptualisation specifying a domain of discourse, is suitable for inclusion in an ontology. An ontology could be considered as an integral part of a declarative knowledge representation language. During the last few years a growing interest in the design, use, and sharing of ontologies has been observed. Work in this area obviously incorporates formal knowledge representation with practical implemented systems. Nowadays, this new emerging field of *ontological engineering* is a craft rather than a science. One reason is that there is no foundation and definition of the main concepts and methodologies that govern the development of ontologies. There is even no clear idea of what an ontology is, see [26].

In the present paper the ontological foundations of modelling, acquisition and representation of knowledge will be discussed. Our approach is guided by some central ideas: the ontological aspects of knowledge, the hierarchical structure of knowledge, and the axiomatic-deductive method.

The paper is organized as follows. Section 2 describes the phases of knowledge acquisition inspired by the axiomatic-deductive method. In section 3 we will discuss the notion of an ontology and its relation to knowledge systems. Section 4 is devoted to the analysis of important ontological basic relations and entities, in particular to the set-mereological aspects of the world. Section 5 describes the first draft of an ontological modelling language extending *KIF* and similar languages such as *F-logic* in a conservative manner. A number of concrete examples are collected section 6.

2 Logical Principles of Knowledge Acquisition

Common-sense knowledge and reasoning is at the center of AI because a human being always starts from a situation in which the information available to him has a common-sense character. Although mathematical models of the traditional kind are contained, at least partially, in common sense, it seems to be impossible to reduce common sense to classical mathematical theories based on set theory. This is for reason of principal: set theory captures only a part of the ontology of the existing world. In spite of this ontological restriction of mathematics, its formal methods present an ideal model for any science, in particular for the evolving science of axiomatic ontology.

2.1 The Axiomatic-Deductive Method

We use the paradigm of the *axiomatic-deductive method* to analyse the main aspects of knowledge processing.

This method contains several principles used for the development of knowledge bases and reasoning systems aiming at the foundation, systematization and formalization of a field of knowledge associated with a given universe of discourse.

The axiomatic method deals with the description of notions and could be motivated by the following considerations. A formal knowledge base includes, on the one hand, entities like primitive notions, defined notions, and definitions, and on the other hand entities like axioms, theorems, and proofs. It would be ideal if one were able to explain explicitly the meaning of every notion occurring in the considered domain and to justifiy each of its propositions immediately. When one tries to explain the meaning of a term, however, one necessarily uses other expressions, and, in turn to explain these expressions, without entering into a vicious circle, one has to resort to further terms again, and so on. We have thus the beginning of a process which can never be brought to an end, a process which can be unterstood as an infinite regress. The situation is quite analogous for the justification of the asserted statements within a knowledge base: for in order to establish the validity of a statement, it is necessary to refer back to other statements, which leads again to an infinite regress.

The axiomatic-deductive method contains the necessary principles to solve this problem. When we set out to construct a given field of knowledge, we distinguish, first of all, a certain small group of concepts in this field that seem to us a priori understandable or intelligible. The expressions of this group we call primitive or basic, and we employ them without formally explaining their meanings via explicite definitions. At the same time we adopt the principle of not employing any of the other terms within the field under consideration, unless their meaning have first been determined with the help of the basis notions and of such expressions of the field whose meaning have been previously explained. The sentence which determines the meaning of a term in this way is called an explicit definition.

How, then, can the basic notions be described, how can their meaning be characterized? From the basic terms, we may construct more complex sentences which may be understood as descriptions of certain formal interrelations between the considered notions. Some of these statements are chosen as axioms; we accept them as true without in any way establishing their validity by means of a proof. By postulating the truth of these sentences we assert that the described interrelations are considered to be valid and at the same time we define the given notions in a certain sense implicitly; i.e. the meaning of the basic terms is captured by the axioms. On the other hand, we agree to accept any other statement as true only if we have succeeded in establishing its validity by admissible deductions. Statements established in this way are called proved statements or theorems.

The method of establishing a field of knowledge in accordance with these principles is called the *axiomatic-deductive method*. An axiomatic-deductive system is a set of propositions in which each proposition is either one of the set of initial propositions (a an axiom or "highest-level hypothesis") or it is a proposition (a "lower-level hypothesis") deduced from the set of initial propositions according to logico-mathematical principles of deduction. In some of the propositions of the system may be propositions about observable concepts (propositions or relations) which are directly testable against experience.

2.2 Phases of Knowledge Acquisition

In this section we will describe in more detail some further aspects of the axiomatic-deductive method. Through which cognitive processes do we create or find the notions and discover the axioms? The process of knowledge acquisition involves several phases.

Identification Phase. In this phase relevant aspects of the problem are uncovered. That means, that takes place the identification of all involved persons and their needs as well as of the problems to be solved and the recources to be used. The following questions must then be answered: How can the problems be characterized? What are the relevant sub-problems and tasks (components)? What do the necessary data look like? Which relations exist between the problem components? The result of this identification phase takes the form of a requirement catalogue for the conceptualisation phase.

Conceptualisation Phase. In the conceptualisation phase it is the following questions which are in the foreground: Which data are available (in contrast to which data are needed)? Which facts are given, which are derived? How can sub-problems be defined and enumerated? In what relationships do the concepts stand? What does the concrete information flow look like? Which processes are necessary for the solution of the problems? In addition, questions about the granularity of knowledge to be represented are to be clarified here. The result of this phase concists in the specification of concepts, e.g. classes and relations. Already in this phase, the development of restricted prototypes for sub-tasks is very helpful. Here an explanation will be able to be obtained if the theoretically developed concepts are practically realizable.

Formalisation and Axiomatization Phase. In the formalisation phase the concepts and relations are expressed in an adequate, *formal framework*. With the selection of a rule-based system the domain expert's assessment should be expressed in IF-THEN rules. An interesting approach is the use of an extended logic programming system to formalize default rules and non-monotonic reasoning, [22]. In our paper we will prefer the logic programming method. The formalisation phase can be compared to the generation of a pseudocode in conventional programming techniques. The result of this phase is a knowledge base representing concepts and rules.

Implementation Phase. In the implementation phase the formalized knowledge is translated into a machine-readable code. The efficiency of the design, developed in the previous phases, can thus be checked through implementation. With high probability the design will require several revisions.

Test Phase. The implementation test phase merges directly into the test phase. Besides the tests of the efficiency of the prototype, the most frequent error source is situated in the IF-THEN rules, which are only in the rarest of cases capable of being considered. The results from the test phase will lead with high probability to several revisions of the design.

The outcome of the first three phases is a set of expressions in a formalized language. These present the knowledge in the form of axioms, typically constituted by rules. This structured system of propositions is called a *formal knowledge base*. By a semantical transformation this knowledge has to be translated into an executable program; this transformation can be understood as a subtask of the implementation phase.

2.3 Levels of a Knowledge Base

A knowledge base usually contains different (sometimes hidden) levels of generality. Thus, a knowledge system will make use of a basic logic which provides the principles of deduction. The basic logic includes all the deductive principles of the system. Thus none of the latter is specific to the system itself and the deductive power of the system will be achieved through the addition of the system's axioms.

The axioms can be classified into three main groups, one group consisting of those axioms required for the basic logic. These logical axioms are true in every possible world. General ontological axioms are concerned with axioms about the ontologically basic relations, which will be studied in section 4. They describe those laws of the ontologically basic relations and entities which are true in every part of the world in which they occur. General ontological axioms present what is sometimes called the *top level ontology.* Finally, domain-specific axioms are tailored to a concrete area of the world. The domain specific knowledge can be refined to further levels, but we consider them, in contrast to the project CYC, as belonging to the same principal level. These three principal levels are natural, the refinement of the domain specific level is rather arbitrary.

In summary, we distinguish the logical level, the general ontological level, and the domain specific level. We give some examples of axioms belonging to different levels of generality.

Example 1 (Logical Axioms) –
- *Logical axioms:*
- $\forall x(P(x) \vee \neg P(x))$
- $\exists x(D(x) \rightarrow \forall y D(y))$
- $\exists x \forall y D(x,y) \rightarrow \forall y \exists x D(x,y).$

Example 2 (General ontological axioms) –
- *Axioms of set theory:*
  $\forall xy \exists z(\forall u(u \in z \longleftrightarrow u = x \ \vee \ u = y).$
- *Part-whole relation*
  $\forall xyz(x \, \mu \, y \wedge y \mu z \rightarrow x \mu z).$
- *Identity*
  $\forall xy(x = y \rightarrow y = x)$
- *Axioms of mereotopology:*
  "Every line is a boundary of a surface"

Example 3 (Domain-specific Axioms) –
- *axioms on genetics,*
- *certain physical laws etc.*

3 Ontologies and Knowledge Systems

In this section we will introduce more precisely the terms 'ontology' and 'conceptualization', which were used in the previous section in an informal manner. The term 'ontology' became popular within the knowledge engineering community with the work of Gruber[27], but its usage has remained unclear. There are several ways to make this notion more precise. We have taken as reference for our analysis the papers [27], [29] and have added some further clarifications.

3.1 The Notion of Ontology

There are several approaches to defining the notion of an ontology. We will discuss those which we will assume in the following. *Aristotle* defined metaphysics (a term corresponding to the term 'ontology') as the science of being as such, as contrasted with the special sciences, each of which investigates a certain class of things and their properties. Ontology considers “all the species of being *qua* being and the attributes which belong to it *qua* being” (*Metaphysics, IV,1*). This level of description we call *General Ontology*, in contrast with the various *Domain Specific, Special* or *Regional Ontologies.*

In the sequel we assume the following definition of ontology as a research area:

> *Ontology is the science which is aimed at the systematic and axiomatic development of the theory of all forms and modes of being, [13].*

Such a science is obviously a foundation for the development of those knowledge bases called “ontologies” in the AI-communitiy. There are several interpretations of this term in use, we have selected some typical examples.

1. An ontology is an explicit specification of a conceptualization.
2. An ontology is a representation of a conceptual system via a logical theory.
3. An (AI)-ontology is a theory of which entities can exist in the mind of a knowledgeable agent.
4. Ontologies are agreements about shared conceptualizations.

The interpretation 1. was proposed in [25] as a definition of what an ontology is for the AI-community. The interpretation 2 is formulated in [29] and is essentially the same as 1. The interpretation 3 refers to ontology as the theory of being from the subjectivistic perspective. The agreements mentioned in interpretation 4 can be understood as specifications of conceptualizations supported and accepted by the majority of the people working in a certain domain.

We assume the point of view of realism, i.e. the position that the kind of things we are speaking about have objective existence, so there are only good conceptualizations. This realism will be further expounded in section 4. We propose in this paper a revised definition of Gruber’s interpretation [25], and reinterpret it from the point of view of the axiomatic method. Our analysis adds in the next paragraph a further clarification to the investigation of Guarino’s [29], where the definition of an “intensional relation” in terms of possible worlds semantics seems to be misleading.

3.2 Conceptualizations

In current practice the term 'ontology' is used ambiguously either to refer to a symbolic system or to its semantical counterpart. In [23] a conceptualization is given by a structure $(D, R_1, \ldots, R_n)$ representing a set D of objects and a number of relations $R_1, \ldots, R_n$. We may use set-theoretical terms to specifiy such systems. Then, all the relations are considered as extensional. Obviously, such a conceptualization is a (somehow) vague description of a part of the world. In which language (apart from natural language) may such a system be specified? We surely need a *conceptualization language* containing set theory and some other fundamental categories. Guarino [29] criticizes Genesereth-Nilsson’s notion of a conceptualization because it refers exclusively to "ordinary mathematical relations on D", i.e. extensional relations. He wants to focus on the *meaning* of these relations instead, independently of a state of affairs; he introduces the term intensional relation or conceptual relation, and tries to capture this notion by using total functions from possible worlds into sets. A conceptualization then is defined as a tuple $\langle D, W, \mathcal{R} \rangle$, where $\mathcal{R}$ is a set of conceptional relations such that every $R \in \mathcal{R}$ is considered as a total function assigning to every $w \in W$ a relation $R(w)$. But, this total function and the relations $R(w)$ are extensional objects themselves and one could doubt whether we gain by this construction some insight in the term "intensional". The point is that not every entity in the world is a set and thus set theory is not able to capture the world's full ontology.

Another point concerns how this conceptualization $\mathcal{C} = \langle D, W, \mathcal{R} \rangle$ is described and specified. In [29] it is implicitly suggested that $\mathcal{C}$ should be specified in set theoretical terms. As an example of such a specification we may consider the field of real numbers which may be defined (up to isomorphism) in set theory. We may guess that in [29] set theory is considered as a kind of modelling language for specifying conceptualizations. The notion of a conceptualization as introduced in [29] can be equivalently decribed by a class of first-order structures of a certain signature satisfying the additional condition that their domains coincide. But, many of these classes cannot be defined by a set-theoretical formula. Thus, the approach in [29] can be summarized as follows. Let there be given a class $\mathcal{K}$ of structures representing our intuitions about a given domain of discourse. We say that a set-theoretical formula φ partially describes $\mathcal{K}$ if $\mathcal{K} \subseteq Mod(\varphi)$, where $Mod(\varphi)$ is the class of structures specified by φ. Therefore, conceptualizations are partially specified by set-theoretical formulas.

In contradistinction to this approach we use a more general ontological language, which includes set theory as a proper part. This modelling language can be used to express partial descriptions of conceptualizations. Furthermore, we use situations instead of first-order structures. "Intensions" then are nothing other than classes of entities whose ontology cannot be fully captured by set theory; we will have to use certain ontologically basic relations and predicates such as inherence, part-whole, causality, time, space, universals, and situations. Some of these relations will be investigated in section 4.

3.3 Formal Knowledge Bases

Every formal knowledge base is built on a conceptualization which is specified by means of general ontological terms such as object, set, relation, process etc. Normally we have to use a formal knowledge representation language to specify the propositions of the knowledge base. There should be also a clear concept of the truth relation defining the meaning of the term " a proposition ϕ is true of the conceptualization", or more generally, a rule stating how the propositions are related to the conceptualization. We adopt a realistic view of the ontology of truth, based on a suitable correspondence theory. On the one hand, we assume that propositions are bearers of the properties of truth and falsehood; on the other hand, we presume the existence of entities in virtue of which propositions are true. These entities we call, as introduced in [37], *truth-makers*, and a conceptualization can be understood as a class of truth-makers. We use here the notion of truth-maker in the broader sense of being a part of the world in which a proposition can be satisfied, i.e. can be made true by certain entities contained in it. A conceptualization, then, can be understood as a class $\mathcal{K}$ of extended truth-makers representing our intuitions about a part of the world.

To specify a conceptualization $\mathcal{K}$ we need ontological relations and entities which cannot be adequately captured by extensional set theoretical notions, for example such entities and relations as "universals", "part-whole", "moments", "substances' etc. Then, the first part of a conceptualization consists of a description of the "signature", i.e. the kinds of relations and entities which we want to describe in the knowledge base. In the second part a sentence ϕ has to be constructed which partially specifies the class $\mathcal{K}$, i.e. the condition $\mathcal{K} \subseteq Spec(\phi)$ should be fullfilled. The knowledge base over this specification is then defined by $\{\psi \mid Spec(\phi) \models \psi\}$. Here, $\models$ is a suitable truth-relation relating propositions to the class $Spec(\phi)$. The simplest form of a truth-relation was formalized by Tarski, [45], but, his correspondence theory is not fully developed because it is restricted to the investigation of set-theoretical surrogates and excludes the real ontology of the world.

An important problem in specifying a knowledge representation language is to develop an ontological fundation which may serve as the language's formal semantics. The current knowledge represention languages are mainly based on classical set theory. This means that the terms used in these languages are reduced to set-theoretical notions. For example, a relation is considered as a set of tuples of objects. It is our aim to develop a language which goes beyond set theory by including further notions that refer to objects in the world which are not sets, for example: trees, houses, water, people, processes, qualities,etc.

4 Ontologically Basic Relations and Entities

In this section we will introduce and discuss the ontologically basic entities and certain basic relations between them. These relations and entities correspond to the most general categories of the world, such as space, time, reality, substance, property, mind, matter, states and the like. It is a permanent concern of philosophers whether these categories are absolute, complete and universal. Aristotle lists ten categories: substance, quantity , quality, relation, place, time, posture, state, action, and passion; but he was not absolutely convinced that the classification was definite. Our analysis is similar to that in [39], but adds the idea of large ontological regions such as the region described by set theory.

Our main distinction is between *Urelemente* and *sets*. We assume the existence of both urelements and sets in the world and presuppose that both the impure sets and the pure sets constructed over the urelements belong to the world. This implies, in particular, that the world is closed under all set-theoretical constructions.

Sets and Urelements. Urelements are entities which are not sets. By a class we understand a collection of entities defined by a condition. This explanation depends on the meaning of the term "condition". Conditions are specified by formulas of a language; in our approach we use a formal language *GOL* (*General Ontological Language*), which is outlined in section 5. Let U be the collection of urelements which is a class and let M be a subclass of U that is also a set. Then we define the cumulative hierarchy $R(M)$ of sets over M in stages in the usual fashion as follows, where On is the class of ordinals.

- $R(M)_0 = M$
- $R(M)_{\alpha+1} = R(M)_\alpha \cup Pow(R(M)_\alpha)$
- $R(M)_\lambda = \bigcup_{\alpha<\lambda} R(M)_\alpha$, λ a limit ordinal.

Neither the membership relation nor the subclass relation can unfold the internal structure of urelements. There arise the following questions:

1. Which categories of urelements are there?
2. Which formal relations hold among the urelements and between the urelements and the non-urelements?
3. How are the formal relations between urelements related to and distinguished from set-theoretical relations?

We classify urelements into individuals and universals, and we classify individuals further into substances, moments and situations. There are two sorts of universals which play an especially central role in formal ontology: time and space, whose instances we call chronoids and topoids, respectively.

At the bottom we have the class U of urelements thought of as a realm of concretely existing things in the world (not sets), within the confines of space and time. What we need is a subcategorization of urelements. We discuss now the intuitions behind the main entities and ontological basic relations.

Individuals and Universals. We shall assume the existence of two main categories of urelements, namely *individuals* and *universals*. An individual is a single thing thought of in contrast to universals. A universal is an entity that can be instantiated by a number of different individuals. The individuals covered by a universal are thus similar in some respect. There are different approaches to universals: *Platonism* is the position that universals exists independently and before individuals (*ante rem*); the *Aristotelian* belief is that universals exist in the individuals (*in re*) and thus not independently from them; *conceptualism* is the view that universals are the reflections of the propensity of the mind to group things together (*post rem*) or that universals are somehow abstracted from individuals. There is also a general suspicion that the whole issue is the result of a misleading reification, trapping us into thinking of two categories of things (the individual and the universal) instead of just individuals. In our approach we assume that the universals exist in the individuals (*in re*) but not independently from them, thus, our attitude is Aristotelian in spirit.

The world is divided into several ontological regions: one of them is the area of sets. For every universal $\mathcal{U}$ there is a class $Ext(\mathcal{U})$ containing all instances of $\mathcal{U}$ as elements. We hold that not every (set-theoretical) class is the extension of a universal. Concerning the regional ontology of sets our position is similar to that of *K. Gödel*,[31]. Gödel believed that concepts and sets represent an aspect of the objective reality. He was even convinced that we have something like a perception of the objects of set theory; we form our ideas also of these objects on the basis of something else which is a priori given. In the sequel it will be important to emphasize that the ontological basic relations and predicates are special classes, but not necessarily extensions of universals; the terms "predicate" and "unary relation" are used synonymously. Thus, we have a predicate $Ur(x) =_{df} x$ is an urelement, and predicates $Ind(x)$, $Univ(x)$ for individuals and universals. Then, we take for granted the axioms that the class of urelements is the disjoint union of the class of individuals and the class of universals, and that there is no entitity being both an individual and a universal. We assume that there are no universals whose extensions are $Ind(x)$ or $Univ(x)$.

The individuals are further subcategorized into *moments, substances, chronoids, topoids* and *situations*.[1] Thus, we get (at least) the following ontological basic predicates: $Mom(x)$, $Subst(x)$, $Sit(x)$, $Chron(x)$, $Top(x)$. We assume, that $Subst(x)$ is the extension of the universal UnS, $Chron(x)$ is the extension of the universal $Time$, and $Top(x)$ is the extension of the universal $Space$.[2] Let us consider substances and moments.

Substances. Firstly, we will review some of the main approaches to the concept of substance.

1) A substance of a thing may be its essence or that which makes it what it is. This will ensure that the substance of a thing is that which remains the same through changes in its accidents. In *Aristotle* (Metaphysics, Z, vii) this essence becomes more than just the matter, but a unity of matter and form.

2) Substance is that which can exist by itself, or does not need a subject in order to exist, in the way that properties need objects; hence a substance is that which bears properties.

The notion of substance tends to disappear in empiristic thought. Metaphysical systems inspired by modern science tend to reject the concept of substance in favour of concepts such as those of field or process. Our starting point is substance in the meaning 2). An individual substance cannot be a bearer of arbitrary properties. By the *appearance* of an individual substance we understand the moments it bears and the relational moments connecting it to other individual substances. We assume the axiom that every individual substance has an (non-empty) appearance. By an object we understand a coherent part of the world that includes an individual substance with its boundary and with some of its moments inhering in it. There seems to be no clear distinction between an object and a situation. We are also confronted with the problem how an individual

[1] This classification is, of course, tentative.

[2] In our approach topoids and chronoids are individuals. This can be assumed consistently if there is an absolute space and an absolute time. Then topoids can be unterstood as individual regions of absolute space. It is our intention to develop a relativistic common sense theory of time and space. Such a theory will be rather complicated and in the present paper we exclude this problem from our considerations.

substance should be structured in such a way as to have a particular appearance. We assume that there is a universal UnS "substance" whose instances are individual substances.

There are branching points for axioms concerning the basic entities and relations. One of the possibilities is to assume the existence of atomic individual substances, i.e. individual substances without substantial parts. In the current paper we make no assumption about the existence of monads and adopt only the axiom that every individual substance has a non-empty appearance.

Moments. The origin of this notion is that of "accident" in Aristotle's *Metaphysics* and *Categories*. Here, an accident is a property of a thing which is not a part of the essence of the thing: something it could lose or have added without ceasing to be the same thing or the same substance. We use the term "moment" in a more general sense and do not distinguish between essential and inessential moments. Moments include individual qualities, actions and passions, a flash, a handshake, thoughts and so on; moments thus comprehend what are sometimes referred to as "events'. The loss of a moment in this more general sense may change the essence of a thing. Moments have in common that they all dependent on individual substances. Let $Mom(x)$ be the ontological basic predicate for moments. We assume that $Mom(x)$ is not the extension of a universal. Moments can be classified with respect to several aspects. The *arity* of a moment is the number of its arguments; unary moments are called *qualities* or *attributes*. *Relational moments* are non-unary moments[3] having a fixed finite number of arguments, and *anadic moments* have a non-determinate number of arguments without fixed upper bound. The basic ontological relation of inherence, denoted by $\succ:$, connects moments with substances. $\rho \succ: (s_1, \ldots, s_n)$ means that the n-ary moment ρ inheres in the substances $s_1, \ldots, s_n$ taking into account the ordering between the $s_i's$ specified by the n-tuple.

Situations and Ontograms. The situation is the most important of the ontological entities. We distinguish elementary and general situations. An *elementary situation* is a part of the world that can be comprehended as a whole; it is composed from individual substances and the relational moments which glue them together. There is a basic predicate $Sit_e(x)$ for elementary situations, and a predicate $Sit(x)$ for general situations. We exclude the existence of universals whose instances are the elementary or general situations. General situations are built up from elementary situations by using further basic ontological relations, as for example causality and intentionality. Ontograms are situations together with a collection of universals.

Our approach differs essentially from that of *Barwise* [5]. Barwise didn't elaborate an ontology for relations; in his theory, in particular, the relation of inherence is missing. The notion of an object is not further analysed either; there is nothing in his theory that corresponds to substances and their appeareances.

[3] In the more developed theory of ontological relations we will expound the thesis that every moment is genuinely unary and that the apparent non-unarity derives from the moment's occurrence in certain situations.

He introduces states of affairs as certain tuples $\langle\langle R, \overline{a}; i\rangle\rangle$, where i is a "polarity" and the polarity 0 indicates something like a negation. There is no convincing ontological interpretation of what it could mean that a state of affairs has polarity 0. He also tends to interpret the relation R in an extensional sense, not as a universal. In our framework the state of affairs $\langle\langle R, \overline{a}; i\rangle\rangle$ with polarity 1 could be read as: there is an instance of the relation R inhering in the tuple $\overline{a}$. Another weakness of his approach is the missing distinction between sets and non-sets, i.e. urelements, representing individuals and universals. This implies, that his situation theory is essentially set theory, with some pseudo-ontological notions added.

Chronoids and Topoids.[4] Chronoids and topoids are instances of the universals time and space. Chronoids can be understood as "durations", and topoids as "space regions" with a certain mereotopological structure. Chronoids and topoids have no independent existence; they depend on situations, since every part of the world is changing and occupies a space region. Our approach to space and time is based on the ideas of *F. Brentano* [9] who developed and elaborated Aristotle's sketchy remarks in the *Physics* about boundaries and continua. *Chisholm* in [11],[12] has made a first step towards interpreting Brentano's idea in a formal manner, and *B. Smith* in [41] has continued and extended this work by presenting a strict axiomatic system about mereotopology.

The standard set-theoretic treatment of time and space is not able to capture the real phenomena of the world. Imagine, for example, a body which is during a certain time interval at rest and then starts to move. Is there a last time point t_1 when the body is at rest and a first time point t_2 when it is in motion? Clearly not, if we use the set-theoretical description of the continuum. The analogous question can be stated for space regions. Consider a closed disc that is divided into two different symmetrical connected subregions, one of which is white, the other red. What happens as we pass along a line which marks the boundary between the two? Do we pass a last point that is white and a first point that is red? These problems can be solved by using an alternative approach to space and time which was expounded by *Chisholm*, [11], [12], and which is based on the ideas of *F. Brentano* in [9]; it is the theory of coincidence of boundaries. *B. Smith*, using these ideas, has outlined in [41] an axiomatic theory of mereotopology which presents an important step in developing a theory of the real ontology of space and time. Our theory of chronoids and topoids will be based on the approach of *Brentano-Chisholm-Smith*.

Ontologically Basic Relations. Up to now, several basic classes of individuals have been considered separately. We will discuss now certain basic ontological relations needed to glue the considered entities together. In the current paper we restrict out intention to the relations of inherence, denoted by $\succ:$, part-whole, denoted by μ, the relativized ternary part-whole relation ν, the instantiation relation, denoted by $::$, the time-frame relation $:\sqsubset_t$, the space framing relation

[4] In the more developed theory we will introduce chrono-topoids which are integrated time-space individuals depending on situations via a framing relation.

$:\sqsubset_s$, the membership relation $\in$, and the containment relation $\rhd$. We continue our discussion with some clarifying remarks about the ontologically basic relations. The term "inherence", "to inhere" can be understood as the translation of the Latin expression "in subjecto esse", in distinction to "de subjecto dici" which may be translated as "to be predicated of a subject" (predication). The inherence relation $\succ:$, sometimes called ontic predication, glues moments to substances. We exclude the possibility that the relation $\succ:$ can be iterated, ie. there are no moments inhering in moments; on the other hand it seems conceivable that there are ontological dependency-relations between moments still not covered by the current relations.

The part-whole relation μ should have an individual as its second argument, i.e. if $a\,\mu\,b$, then b is an individual.[5] From this follows that sets don't have parts; on the other hand, sets can be parts of individuals. We will discuss the strong axiom that every set whose elements are parts of a certain individual is itself a part of that individual. The ternary part-whole relation $\nu(x,y,z)$ has the meaning: "z is a universal and x is a part of y *in the sense of* z". Obviously, $\nu(x,y,z)$ implies $x\,\mu\,y$, but not conversely. These universal-dependent part-whole relations are very important in applications.

The symbol :: denotes the instantiation relation, its first argument is an individual, and its second a universal. The instances of a universal u are "individualizations" which have a part in u. A universal is an entity which captures something general that is realized in the individuals. In the present stage of our theory we exclude the possiblity that universals can be instances of meta-universals (so-called second intentions). The meta-universals which are needed in applications are presented by classes whose elements are universals. Let us consider this point in more detail. To every formula $F(x)$ there corresponds a class, namely, the collection of all entities satisfying $F(x)$. This class is denoted by $\{x \mid F(x)\}$ and shorter by λxF. Then, the expression $a \in \lambda xF$ means that a belongs to the collection λxF. It seems to be that such classes are sufficient to model meta-universals.

The binary relation $:\sqsubset_t$ glues chronoids to situations. We presume that every situation is framed by a chronoid and that every chronoid frames a situation. The relation $x :\sqsubset_t y$ is to be read "the chronoid x frames the situation y". Let s be a situation, then $chr(s)$ denotes the chronoid framing s. For every part t of $chr(s)$ we may define the temporal projection $s \downarrow t$ from s to t. This projection is not neccessarily stable with respect to a universal u, i.e. it may happen that every instance of u is a situation whose proper temporal projection does not instantiate u. Analogously, the binary relation $:\sqsubset_s$ glues topoids to situations. Again, we presume that every situation is framed by a topoid and that every topoid frames a situation. The relation $x :\sqsubset_s y$ is to be read "the topoid x frames the situation y".

The binary relation $x \bot y$ is to be read "x is an individual substance belonging to the situation y", and $x \top y$ means "x is a moment inhering in a substance belonging to the situation s". Obviously, the relation $x \top y$ may be defined by

[5] A possible part-whole relation for universals will be something completely different.

the following formula: $x \top y =_{df} Mom(x) \wedge Sit(y) \wedge \exists z (z \bot y \wedge x \succ\colon z)$. Finally, we have the relation $x \rhd y$ with the meaning: "x is an individual, y is a situation, and x is contained in y or belongs to y". Which individuals belong to a situation s? All substances and its moments appearing in s belong to s, all parts of s, the chronoid of s and its parts, etc.

5 Outline of a Modelling Language

In this section we will outline a modelling language using the concepts which were expounded in the preceding sections.

5.1 Syntax and Axioms

Our intended modelling language GOL (General Ontological Language) is formalized in a first-order language.[6] We presume a basic ontological vocabulary, denoted by Bas, containing the following groups of symbols, where $R(x)$ denotes the symbol R with argument x.

Unary relational symbols.

- $Ur(x)$ (urelement)
- $Set(x)$ (set)
- $Ind(x)$ (individual)
- $Univ(x)$ (universal)
- $Mom(x)$ (moment)
- $Sit(x)$ (general situation)
- $Sitel(x)$ (elementary situation)
- $Chron(x)$ (chronoid)
- $Top(x)$ (topoid)
- $Subst(x)$ (individual substance)

Symbols for universals.

- UnS (the universal "substance")
- $Time$ (time)
- $Space$ (space)

Symbols for binary relations.

- $\in$ (membership)
- $::$ (instantiation)
- $\succ\colon$ (inherence)
- μ (part-whole)

[6] There is nothing surprising in our use a first-order language for the formalization of ontological categories. The point is that the semantics of the ontology is captured by the axioms assumed to be true about the considered entities. In particular, we include the first-order axiomatization ZF of set theory in our language.

- $\nu(x, y, z)$ (relativized part-whole)
- $\perp$ (is substance in)
- $\top$ (is moment in)
- $:\sqsubset_t$ (time framing)
- $:\sqsubset_s$ (space framing)
- $=$ (equality)
- $\triangleright$ (is contained in)

To the basic vocabulary we may add further symbols used for domain specific areas; the domain specific vocabulary is called an *ontological signature*, denoted by Σ. An *ontological signature* Σ is determined

- by a set ExtR of symbols used to denote extensional relations,
- by a set U of symbols used to denote universals,
- by a set K of symbols used to denote individuals.

An ontological signature is summarized by a tuple $(\mathsf{ExtR}\ ,\mathsf{U},\mathsf{K};ar)$, where ar is an arity function $ar : \mathsf{ExtR} \cup \mathsf{U} \to \omega \cup \{\infty\}$. $ar(r)$ denotes the arity of the symbol r, where $ar(r) \in \omega$ if $r \in \mathsf{ExtR}$. $ar(u) = \infty$ means that the symbol u is anadic.

The syntax of the language $GOL(\Sigma)$ is defined by the set of all expressions containing the atomic formulas and closed with respect to the application of the logical functors $\vee, \wedge, \to, \neg, \leftrightarrow$, and the quantifiers $\forall, \exists$. We use untyped variables $x, y, z, \ldots$; terms are variables or elements from $\mathsf{U} \cup \mathsf{K} \cup \{UnS, Time, Space\}$.

Atomic formulas have the form:

- $t = s$, $t \in s$ for terms s,t
- $v \succ: (s_1, \ldots, s_n)$ for terms v, s_i
- $v :: u$ for $u \in \mathsf{U}$ and $ar(u) = n$, $t :: u$,
- $t :\sqsubset_s s, t :\sqsubset_t s$,
- $t \triangleright s$, t a term, s a situation
- $R(t_1, \ldots, t_n)$ for $R \in \mathsf{ExtR}$.

$FmGOL(\Sigma)$ is the smallest set containing the atomic formulas $AtGOL(\Sigma)$ and closed w.r.t. $\wedge, \vee, \to, \neg, \leftrightarrow, \forall, \exists$.

The language includes an axiomatization capturing the semantics of the ontologically basic relations. We don't present the axiomatization in full detail, but illustrate the main groups by selecting some typical axioms. We introduce three groups of axioms whose union are the axioms $Ax(GOL)$ for the language GOL. Let be $Ax(GOL) =$ Logical axioms $\cup$ Axioms about Bas $\cup$ Axioms about Σ.

I. Logical Axioms[7]

(a) Axioms of propositional logic (Frege's system)

1. $A \to (B \to A)$

[7] There are many equivalent systems, we present here a particularly simple one.

2. $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$
3. $(A \to (B \to C)) \to (B \to (A \to C))$
4. $(A \to B) \to (\neg B \to \neg A)$
5. $\neg\neg A \to A$
6. $A \to \neg\neg A$

The other functors are introduced by definitions:

1. $A \wedge B =_{df} \neg(A \to \neg B)$
2. $A \vee B =_{df} \neg A \to B$
3. $A \leftrightarrow B =_{df} \neg((A \to B) \to \neg(B \to A))$

(b) Axioms of predicate logic

1. $\forall x(A \to B) \to (\forall x A \to B)$
2. $\forall x(A \to B) \to (A \to \forall x B)$, x not free in A
3. $\forall x A \to A(x/t)$, t is a term being free for x
4. $\exists x A(x) \leftrightarrow \neg\forall x \neg A(x)$

(c) Axioms of identity

1. $\forall x(x = x)$
2. $\forall xy(x = y \to y = x)$
3. $\forall xyz(x = y \wedge y = z \to x = z)$
4. $\forall x_1 x_2 \ldots x_n y(x_k = y \to$
   $F(x_1, x_2, \ldots, x_{k-1}, x_k, \ldots, x_n) \leftrightarrow F(x_1, x_2, \ldots, x_{k-1}, y, \ldots, x_n))$,
   for every formula F.

II. Axioms about Basic Ontology

(a) Sort and Existence Axioms

1. $\exists x(Set(x))$,
2. $\exists x(Ur(x))$
3. $\forall x(Set(x) \vee Ur(x))$
4. $\neg\exists x(Set(x) \wedge Ur(x))$
5. $\forall x(Ur(x) \leftrightarrow Ind(x) \vee Univ(x))$
6. $\neg\exists x(Ind(x) \wedge Univ(x))$

(b) Axioms about sets including ZF

1. $\forall uv\exists x(Set(x) \wedge x = \{u, v\})$
2. $\{\phi^{Set} \mid \phi \in ZF\}$, where ϕ^{Set} is the relativization
   of the formula ϕ to the basic symbol $Set(x)$.

(c) Axioms about Moments and Substances

1. $\forall x(Subst(x) \to \exists y(Mom(y) \wedge y \succ\!: x))$

2. $\forall x(Mom(x) \to \exists y(Subst(y) \wedge x \succ: y))$[8]
3. $\forall xyz(Mom(x) \wedge x \succ: y \wedge x \succ z \to y = z)$
4. $\forall xy(x \bot y \to Sit(y) \wedge Subst(x))$
5. $\forall xy(x \top y \to Mom(x) \wedge Sit(y))$

(d) Axioms about Part-Whole

Definitions.

- $o(x, y) =_{df} \exists z(z\, \overline{\mu}\, x \wedge z\, \overline{\mu}\, y)$, (overlap)
- $x\, \overline{\mu}\, y =_{df} x = y \vee x\, \mu\, y$ (reflexive part-whole).

1. $\forall xy(x\, \mu\, y \to Ind(y) \wedge (Ind(x) \vee Set(x))$
2. $\forall xy(Set(x) \wedge \forall z(z \in x \to z\, \mu\, y) \to x \mu y))$
3. $\forall x(\neg x\, \mu\, x)$
4. $\forall xyz(x\, \mu\, y \wedge y\, \mu\, z \to x\, \mu\, z)$
5. $\forall xy(\forall z(z\, \mu\, x \to ov(z, y)) \to x\, \mu\, y))$
6. $\forall xyz(\nu(x, y, z) \to Univ(z) \wedge x\, \mu\, y)$
7. $\forall xyzu(\nu(x, y, u) \wedge \nu(y, z, u) \to \nu(x, z, u))$

(e) Axioms about chronoids and topoids.

Topoids are understood as three-dimensional space regions, and chronoids as durations.

1. $\forall x(Sit(x) \to \exists t(Chron(t) \wedge t :\sqsubset_t x))$
2. $\forall x(Sit(x) \to \exists t(Top(t) \wedge t :\sqsubset_s x))$
3. $\forall x(Chron(x) \to \exists x_1 \exists s(Chron(x_1) \wedge Sit(s) \wedge x\, \overline{\mu}\, x_1 \wedge x_1 :\sqsubset_t s))$
4. $\forall x(Top(x) \to \exists x_1 \exists s(Top(x_1) \wedge Sit(s) \wedge x\, \overline{\mu}\, x_1 \wedge x_1 :\sqsubset_s s))$

(f) Axioms about situations

Definitions.

- $Cont\ (s) =_{df} \{m \mid m \rhd s\}$
- $s \sqsubseteq t =_{df} Cont(s) \subseteq Cont(t)$.

1. $\forall x(Sit(x) \to \exists s \exists t(Subst(s) \wedge Mom(t) \wedge s \rhd x \wedge t \rhd x))$
2. $\forall x(Mom(x) \to \exists s(Sit(s) \wedge x \rhd s))$
3. $\forall x(Subst(x) \to \exists s(Sit(s) \wedge x \rhd s))$
4. $\forall xy(Sit(x) \wedge Sit(y) \to \exists z(Sit(z) \wedge x \sqsubseteq z \wedge y \sqsubseteq z))$
5. $\neg \exists x(Sit(x) \wedge \forall y(Sit(y) \to y \sqsubseteq x))$

III. Axioms about Σ

[8] A refined theory of moments has to be elaborated; in the present axiom the moment is unary. To cover n-ary moments one could introduce predicates $Mom_n(x)$ indicating that the moment is n-ary. The associated axiom takes then the following form: $\forall x(Mom_n(x) \to \exists y_1 \dots y_n(\bigwedge_{1 \le i \le n} Subst(y_i) \wedge x \succ: (y_1, \dots, y_n))$. Up to now, it is not still clear, how to formulate this axiom for anadic moments.

1. $Univ(U)$ for every $U \in \mathsf{U}$
2. $Set(R)$ for every $R \in \mathsf{ExtR}$
3. $Ind(c)$ for every $c \in \mathsf{K}$

A knowledge base about a specific domain w.r.t. the signature Σ is determined by a set of formulas from $GOL(\Sigma)$ which are not basic axioms.

5.2 Rules

As in the modelling language *KIF*, [20], we will introduce additional formal expressions, called rules. These rules can be used to formalize non-monotonic reasoning and to develop deductive databases. This sub-language of GOL to be described is a logical programming language called *KR1-Prolog*. In the present paper, we assume that the signature $\Sigma = \langle \mathsf{ExtR}, \mathsf{K} \rangle$ of this sublanguage contains only extensional relation symbols and constants. [9] In knowledge representation, two different notions of falsity arise in a natural way. Certain facts are *implicitly false by default* in virtue of the fact that they are not true in an intended model of the knowledge base. Others are *explicitly false* by virtue of a direct proof of their falsity, corresponding to their falsification in all intended models. These two kinds of falsity in knowledge representation are captured by the two negations, of partial logic,[33], called *weak*, denoted by $-$, and *strong*, denoted by $\sim$.

U_σ denotes the set of all ground terms of σ. $L(\sigma)$ is the smallest set of expressions containing the atomic formulas of σ, and being closed with respect to the following conditions: if $F, G \in L(\sigma)$, then $\{-F, \sim F, F \wedge G, F \vee G\} \subseteq L(\sigma)$. $L^0(\sigma)$ denotes the corresponding set of sentences (closed formulas). For sublanguages of $L(\sigma)$ formed by means of a subset $\mathcal{F}$ of the logical functors, we will write $L(\sigma; \mathcal{F})$. With respect to a signature σ we define the following sublanguages: $\mathrm{At}(\sigma) = L(\sigma; \emptyset)$, the set of all atomic formulas (also called *atoms*); $\mathrm{Lit}(\sigma) = \mathrm{At}(\sigma) \cup \{\sim a : a \in At(\sigma)\}$, the set of all *literals*, and $\mathrm{XLit}(\sigma) = L(\sigma; -, \sim)$, the set of all *extended literals*. We introduce the following conventions. If $L \subseteq L(\sigma)$ is some sublanguage, L^0 denotes the corresponding set of sentences. If the signature σ does not matter, we will omit it and write, e.g., L instead of $L(\sigma)$. If $\langle Y, < \rangle$ is a partial order, then $\mathrm{Min}(\langle Y, < \rangle)$ denotes the set of all minimal elements of Y, i.e. $\mathrm{Min}(\langle Y, < \rangle) = \{X \in Y \mid \text{ there is no } X' \in Y \text{s.th. } X' < X\}$. $Pow(X)$ or 2^X denote the power set of the set X.

In *KR1* we use sequents for the purpose of representing rule knowledge. A sequent, then, can be considered as a derivation rule for generating knowledge. A *sequent* s is an expression of the form $F_1, \ldots, F_m \Rightarrow G_1, \ldots, G_n$, where $F_i, G_j \in L(\sigma)$ for $i = 1, \ldots, m$ and $j = 1, \ldots, n$, $n, m \geq 0$, and $\Rightarrow$ being the empty sequence for $m = n = 0$. The *body* of s, denoted by Bs, is given by $\{F_1, \ldots, F_m\}$, and the *head* of s, denoted by Hs, is given by $\{G_1, \ldots, G_n\}$. $\mathrm{Seq}(\sigma)$ denotes the class of all sequents s such that $Hs, Bs \subseteq L(\sigma)$, and for a given set $S \subseteq \mathrm{Seq}(\sigma)$, $[S]$ denotes the set of all ground instances of sequents from S. A KR-program, or an extended generalized program, is a set of sequents, and $EGLP$ denotes the set of all extended generalized logic programs.

[9] In a future paper we will extend the rules to the full ontological signature.

The basic semantics of rules is given by *abstract extensional situations (a.e. situations)*. An a.e. situation over σ is a consistent subset I of $\mathrm{Lit}^0(\sigma)$, i.e.a subset containing no ground atom a such that $\{a, \sim a\} \subseteq I$. Let $\boldsymbol{I}(\sigma)$ be the set of all a.e. situations over Σ. For a sequent $\Rightarrow F$ with empty body we also write more simply F. The model relation $\models \subseteq \boldsymbol{I}(\sigma) \times L^0(\sigma)$ between an interpretation and a sentence is defined inductively as follows.

- For literals $l \in \mathrm{Lit}^0(\sigma)$, $I \models l$ iff $l \in I$.
- $I \models F \wedge G$ iff $I \models F$ and $I \models G$.
- $I \models F \vee G$ iff $I \models F$ or $I \models G$.
- $I \models -F$ iff $I \not\models F$.

In addition, we assume DeMorgan-style rewrite rules for handling the combination of $\sim$ with $-, \wedge, \vee$. The model relation between an interpretation $I \in \boldsymbol{I}(\sigma)$ and a formula $F \in L(\sigma)$ is defined by $I \models F$ iff $I \models F\nu$ for every instantiation $\nu : \mathit{Var} \to U_\sigma$. We obtain the model operators $\mathrm{Mod}(X) = \{I \in \boldsymbol{I} : I \models X\}$, and the corresponding consequence relations defined by $X \models F$ iff $\mathrm{Mod}(X) \subseteq \mathrm{Mod}(F)$.

The truth relation $\models$ is extended to rules as follows. Let $I \in \boldsymbol{I}(\sigma)$, $F \in L(\sigma)$, and $\mathrm{Sat}_I(F) = \{\nu \in {U_\sigma}^{\mathit{Var}} : I \models F\nu\}$ Then, $I \models F_1, \ldots, F_m \Rightarrow G_1, \ldots, G_n$ iff $\bigcap_{i \leq m} Sat_I(F_i) \subseteq \bigcup_{j \leq n} Sat_I(G_j)$. For $S \subseteq \mathrm{EGLP}(\sigma)$, we define the model operator $\mathrm{Mod}(S) = \{I \in \boldsymbol{I}(\sigma) : I \models s$, for all $s \in S\}$, and the associated entailment relation $S \models F$ iff $\mathrm{Mod}(S) \subseteq \mathrm{Mod}(F)$, where $F \in L(\sigma)$. Instead of sequent notation $B \Rightarrow H$ we shall also use sometimes the rule notation $F \leftarrow G$, where $F = \bigvee H$ and $G = \bigwedge B$.

For a class of a.e. situations $\boldsymbol{K}$, we write $\boldsymbol{K} \models F$ iff $I \models F$ for all $I \in \boldsymbol{K}$. We denote the set $S_{\mathbf{K}}$ of all sequents from a sequent set S which are applicable in $\boldsymbol{K}$ by $S_{\mathbf{K}} = \{s \in [S] : \boldsymbol{K} \models Bs\}$. If $\boldsymbol{K}$ is a singleton, we omit brackets. For two a.e. situations I, J let be $[I, J] = \{K \mid I \subseteq K \subseteq J\}$.

A semantics for sequents is given by a preferred model operator $\Phi : \mathit{Seq} \to 2^{\boldsymbol{I}}$, satisfying the condition $\Phi(S) \subseteq \mathrm{Mod}(S)$, and defining the preferential entailment relation $S \models_\Phi F$ iff $\Phi(S) \subseteq \mathrm{Mod}(F$. The main question concerning the semantics of logic programs is: which preferred model operator does capture the intended semantics for (generalized) extended logic programs ? The following definition of a *stable generated model* of extended generalized logic programs generalizes the *answer set* semantics of [21], and the semantics introduced in [32].

Let $S \subseteq \mathit{EGLP}(\sigma)$. $M \in \mathrm{Mod}(S)$ is called a *stable generated model* of S, symbolically $M \in \mathrm{Mod}^s(S)$, if there is a chain of Herbrand interpretations $\{I_n \mid n < \omega\}$ such that $\mathit{sup}_{n<\omega} I_n = M$, and

- $I_0 = \emptyset$, and $m \leq n$ implies $I_m \subseteq I_n$, $n < \omega$,
- For every number n I_{n+1} is a minimal extension of I_n satisfying the heads of all sequents whose bodies hold in $[I_n, M]$, i.e.
  $I_{n+1} \in \mathrm{Min}\{I \in \boldsymbol{I}(\sigma) : I \supseteq I_n$, and $I \models \bigvee Hs$, f.a. $s \in S_{[I_n, M]}\}$.

We also say that M is *generated* by the *S-stable chain* $\{I_n \mid n < \omega\}$. The stable entailment relation is defined as follows: $S \models^s F$ iff $\mathrm{Mod}^s(S) \subseteq \mathrm{Mod}(F$, where $F \in L(\sigma)$. In [32] it is shown that stable generated models of disjunctive programs are not always minimal and that there are stable generated minimal models of disjunctive programs which are not fixpoints of the Gelfond-Lifschitz operator. The language *KR1* with the semantics of generated stable models can be used to model several forms of non-monotonic reasoning, [17]; as already shown in [22], this concept was proven to be fruitful in practical applications.

5.3 Semantics of GOL

Semantics, the study of linguistic meaning, is a notoriously difficult topic. As indicated in section 3.3 we assume a realistic approach to meaning using a correspondence theory of truth in the spirit of [37]. In this section we will only sketch some ideas whose elaboration will be a task for future research.

Let Σ be an ontological signature. An abstract Σ-interpretation is a first-order structure $\mathcal{W} = (W, \mathsf{Bas}^\delta, \Sigma^\delta)$, where W is a set and $\Sigma^\delta \cup \mathsf{Bas}^\delta$ are interpretations of the symbols from $\Sigma \cup \mathsf{Bas}$ in the set W. Furthermore, $\mathcal{W}$ satisfies the axioms $Ax(GOL)$. On the other hand, there is the real world $\mathcal{R}$ which is not a set; $\mathcal{W}$ can be understood as a set-theoretical reflection of $\mathcal{R}$. Extended truth-makers $\mathcal{T}$ are parts of the world $\mathcal{R}$ within which formulas from $L(GOL)$ may be satisfied. Then, $\mathcal{T} \models \phi$ means that the sentence $\phi \in L(GOL)$ is true in $\mathcal{T}$.

We restrict our consideration in the following to a special class of extended truth-makers, called ontograms.. An ontogram $\mathcal{T}$ is a part of the world containing a situation and a collection of universals, symbolically $(S, U_1, \ldots, U_n)$; here, we assume that the universals U_i are not vacuous with respect to the situation S, i.e. there are individuals belonging to the situation instantiating the universals $(S, U_1, \ldots, U_n)$. We assume that the containment relation $\rhd$ is defined for ontograms; then let be $Cont(\mathcal{T}) = \{m \mid m \rhd \mathcal{T}\}$. For an ontogram $\mathcal{T}= (S, U_1, \ldots, U_n)$ the containment closure is defined by the following system $\mathcal{C}(\mathcal{T}) = (Cont(\mathcal{T}), S, U_1, \ldots, U_n, \mathsf{Bas} \downarrow Cont(\mathcal{T}))$; here $\mathsf{Bas} \downarrow Cont(\mathcal{T}))$ the restriction of the relations from Bas to the class $Cont(\mathcal{T})$.

Let $\phi \in GOL$ be an arbitrary formula and $\mathcal{T}$ be an ontogram. A function ν is an anchor for ϕ in $\mathcal{T}$ if ν associates to every symbol in $sign(\phi)$ an element from $Cont(\mathcal{T})$. Here, $sign(\phi)$ is the set of all free variables in ϕ and all symbols denoting contants, universals or sets. The function ν has to preserve the type of the symbol. We may then define, using Tarski's definition of truth, the notion "The anchor ν satisfies the sentence ϕ in $\mathcal{C}(\mathcal{T})$", which is denoted by $\mathcal{T} \models_\nu \phi$. This semantics of GOL can be used to give meanings to natural language sentences. Firstly, we will translate a natural language sentence ϕ into an expression $tr(\phi)$ of $L(GOL)$ and then we will interpret the formula $tr(\phi)$ using the sketched semantics for $L(GOL)$. We demonstrate this idea by an example.

Let us consider the sentence $\phi =$ "John is kissing Mary". The words "John" and "Mary" denote individuals "J" and "M". U_{KE} denotes a universale "kissing-event". A suitable translation $tr(\phi)$ of ϕ to GOL gives the following sentence: $\exists x(x :: u_{KE} \wedge x \succ: (j, m))$, where the words "John" and "Mary" are replaced by the individual constants "j" and "m". Then, $tr(\phi)$ is satisfiable in a certain ontogram $\mathcal{T}$ iff there is an anchor ν for $tr(\phi)$ in $\mathcal{T}$ such that $\mathcal{T} \models_\nu tr(\phi)$. The *extensional meaning* of a natural sentence ϕ of this simple form could be defined as the class of all ontograms, denoted by $Ont(\phi)$, in which $tr(\phi)$ is satisfiable.

A more developed linguistic theory has to make further distinctions and has to introduce additional ontologically basic relations. Let ψ be a sentence like "A man kisses a woman twice". Then we introduce the *sentence type* $U(\psi)$ denoting a universal. The instances of $U(\psi)$ are the individual graphic or acoustic occurrences of the sentence ψ. For example, we can formalize "John says (writes) that ψ" by $\exists rs(r :: U(saying) \wedge doing(J,r) \wedge s :: U(\psi) \wedge intend(r,s))$. Next we consider the *thought* (*thought type*) associated with ψ, denoted by $U(<\psi>)$ which is a universal whose instances are certain mental events. Thus "It is thought that ψ" may be formalized by $\exists t(t :: U(<\psi>))$. Finally, there is a universal "state of affair" $U([\psi])$ associated with ψ. The instances of $U([\psi])$ are *individual* state of affairs being contained in a situation. "John sees that ψ" can be formalized as $\exists u, s(u :: U(seeing) \wedge doing(J,u) \wedge s :: U([\psi]) \wedge intend(u,s))$. Here, *doing (x,y), intend(x,y)* are two new ontologically basic relations.

5.4 Comparison to other Languages

In this section we compare our approach to that of *KIF*, [20] *F-logic*, [34], and the family of description logics. It turns out that these language are rather weak because they are based on set theory, which is crippled by its extensionalism.

Knowledge Interchange Format. *Knowledge Interchange Format* (*KIF*) is a formal language for the interchange of knowledge among computer programs, written by different programmers, at different times, in different languages. *KIF* can be considered as a lower-level knowledge modelling language; when a program reads a knowledge base in *KIF*, it converts the data into its own internal form; when the program needs to communicate with another program, it maps its internal data structures into *KIF*.

KIF has the following essential features. The language has a declarative semantics. It is possible to understand the meaning of expressions in the language without appeal to an interpreter for manipulating those expressions. In this way, *KIF* differs from other languages that are based on specific interpreters, such as Emycin and Prolog. The language provides for the expression of arbitrary sentences in predicate calculus. Hence, it differs from relational database languages (many of which are confined to ground atomic sentences) and Prolog-like languages (that are confined to Horn clauses). The language provides for the representation of knowledge about the representation of knowledge. This allows to make all knowledge representation decisions explicit and permits to introduce new knowledge representation constructs without changing the language.

The ontological basis of *KIF* can be extracted from [20]; we summarize the main points. The most general ontological entity in *KIF* is an *object*. The notion of an object, used in *KIF*, is quite broad: objects can be concrete (e.g. a specific carbon, Nietzsche, the moon) or abstract (the concepts of justice, the number two); objects can be primitive or composite, and even fictional (e.g. a unicorn). In *GOL*, in contrast to *KIF*, there is an ontological classification of the basic entitities. In *KIF*, a fundamental distinction is drawn between *individuals* and *sets*. A set is a collection of objects; an individual is any object that is not a set. This distinction corresponds in *GOL* to the difference between urelements and sets. *KIF* adopts a version of the Neumann-Bernays-Gödel set theory, *GOL* assumes *ZF* set theory; but this difference is not essential. The functions and relations in *KIF* are introduced as sets of finite lists; here the term "set" corresponds to the term "class". Obviously, the relations and functions in *KIF* correspond in *GOL* to the *extensional relations*. *KIF* does not provide ontologically basic relations like our inherence, part-whole and the like. Hence, the ontological basis of *KIF* is much weaker than that of *GOL*. *GOL* can be considered as a proper extension of *KIF*; *KIF* can be understood as the extensional part of *GOL*.

Frame-Logic. The term "object-oriented approach", is only a loosely defined, and comprehends a number of notions, such as complex objects, object identity, methods, encapsulation, typing and inheritance, which have been identified as its most important features. One of the main problems with the object-oriented approach is the lack of a logical semantics. Frame-logic is a language that accounts in a declarative fashion for most of the structural aspects of object-oriented and frame-based languages. Furthermore, it is suitable for defining, querying, and manipulating database schemata. *F-logic* has a model-theoretic semantics and a sound and complete proof theory. *F-logic* stands in the same relationship to the object-oriented paradigm as classical predicate calculus stands to relational programming. The ontological basis of this language is purely set-theoretical. The instantiation relation, denoted by :, is modeled by the membership relation, the is-a-relation :: can be explicitly defined in terms of :. The ontological basis seems to be even weaker than that of *KIF*, because the full *ZF* or *GB*-system is not available. Similarly as for *KIF*, *F-logic* captures (some) extensional aspects of *GOL*.

Description Logic. Description logics are specialized languages related to the *KL-ONE* system of Brachman and Schmolze [8]. They are designed for representing knowledge, and the general aim is to provide a small set of operations to describe pieces of information, together with efficient methods to make inferences. Description logics are generally considered to be variations of first-order logic - either restrictions or restrictions plus some added operators. These variations are motivated by the undecidability of the inference problem for first order logic and by the intention to preserve the structure of knowledge to be represented.

The ontological basis of description logics is again set theory, in particular the semantics of first-order predicate calculus. But, in addition, as formulated in [35], the language has to be restricted to formulas of a certain form. This philosophy behind this is called in [16] the *restricted language thesis*. One argument in [35] is that general-purpose knowledge representation systems should restrict their languages by omitting constructs which require non-polynomial (or otherwise unacceptably long) worst-case response time for the correct classification of concepts.

In [16] the position is taken that the restricted language assumptions are flawed. In wider practice the terminological facilities of such systems are so impoverished that the very purpose of general-purpose representational utilities is defeated. In [16] a list of representative examples is expounded that show that many important classes of definable concepts are inexpressible in the restricted languages of *KL-ONE* and its descendents. These restrictions severely impair the utility for representing and modelling knowledge.

CycL. *CycL* was developed by Lenat and Guha [15] for the specification of common-sense ontologies. The semantics of the formal language *Cycl* is based exclusively on set theory, as *KIF* and F-logic. There are some superflous notions for example four existential quantifiers which may, however, be reduced by explicite definitions to one existential quantifier. The second-order quantification over predicates and functions[10] captures only a restricted part of *ZF* (which is used in *GOL*) or *GB-class theory* (which is used in *KIF*). Hence, the system *CycL* is weaker than *KIF* and the extensional part of *GOL*.

6 Applications to Knowledge Modelling

In this section we collect several examples to explain, demonstrate and clarify the previous concepts. We select areas of different kinds.

6.1 Classical Geometry

Firstly, we consider elementary geometry as introduced by A. Tarski,[46]. The entities to be investigated are points. Two extensional relations $\beta(x,y,z)$ and $\delta(x,y,u,v)$ are used denoting the betweenness relation and the equidistance relation, i.e. the formula $\beta(x,y,z)$ is read "y lies between x and y", and the formula $\delta(x,y,u,v)$ is read "x is as distant from y as v is from u". To simplify, we may envisage the two-dimensional Euclidean plane to in order to gain an understanding of points and the relations β and δ. Obviously, points and atomic topoids are distinct entities. Tarksi's elementary geometry $\mathcal{G}_T$ contains 12 axioms and one schema (the continuity schema). We will select some of them for demonstration.

1. $\forall xy(\beta(x,y,z) \rightarrow x = y)$
   (identity axiom for betweeness)
2. $\forall xyzu(\beta(x,y,u) \wedge \beta(y,z,u) \rightarrow \beta(x,y,z))$
   (transitivity for betweeness)

[10] There seems to be a confusion about the term "reification". In most cases this term means to consider an entity as an element of a set, nothing more. But, the claim that such a procedure generates new individuals (in proper sense) is a fraud.

3. $\forall xyzu(\beta(x,y,z) \wedge \beta(x,y,u) \wedge x \neq y \rightarrow \beta(x,z,u) \vee \beta(x,uz))$
   (connectivity for betweeness)
4. $\forall xy(\delta(x,y,y,x)$
   (reflexivity for equidistance)
5. $\forall xyz(\delta(x,y,z,z) \rightarrow x = y)$
   (identity axiom for equidistance)
6. $\forall xyzuvw(\delta(x,y,z,u) \wedge \delta(x,y,v,w) \rightarrow \delta(z,u,v,w))$
   (transitivity for equidistance)
7. $\forall xyzut\exists v(\beta(x,t,u) \wedge \beta(y,u,z) \rightarrow \beta(x,v,y) \wedge \beta(z,t,v))$
   (Pasch's axiom)
8. $\forall txyzu\exists vw(\beta(x,u,t) \wedge \beta(y,u,z) \wedge x \neq u \rightarrow \beta(x,z,v) \wedge \beta(x,y,w) \wedge \beta(v,t,w))$
   (Euclid's axiom)

Since β, δ are extensional relations, i.e. sets, only sets can be used as interpretations of $\mathcal{G}_T$. By a model of $\mathcal{G}_T$ is understood a triple $\mathcal{M} = (A, B, D)$, where A is a set, representing the points, a ternary relation B for betweenness, and a quaternary relation D interpreting equidistance. The most familiar examples of models are certain Cartesian spaces over ordered fields. We want to emphasize that this geometry is interpreted in the regional ontology of sets. On the other hand, topoids represent the real geometric entities (which are not sets) and we may suggest the definition: real geometry is the ontological theory of topoids. Let us consider a particular question: assume that the physical universe P is a situation, then it is framed by a topoid τ. Which axioms do we need to describe the mereo topological stucture of τ? Let us take for granted that we have already a theory $\mathcal{G}_\tau$. How, then, do the theories $\mathcal{G}_T$ and $\mathcal{G}_\tau$ relate to each other? To what extent and in which sense does Tarksi's geometry capture properties of the real geometry of τ?

6.2 Mereotopology and Morphology

There are several approaches to the ontology of space; most of them are based on the conviction that the mathematical standard treatment using point-set topology is not sufficient to capture real world phenomena. Mereology was developed as an alternative to set theory, and the part-whole relation μ is one of the ontological basic relations needed to describe the world. But a purely mereological outlook is too narrow to describe spatial entities and the structure of spatial localizations. In Varzi[47] several strategies are expounded to cope with this problem of expressivity; we take here the most obvious approach, namely to add further space-relevant basic relations to the mereological basic relation μ. Topology provides a natural next step after mereology in the development of a comprehensive part-whole theory; thus a theory of parts and wholes needs to incorporate a topological machinery of some sort. Such needs will be obvious, especially so in connection with qualitative reasoning about space and time; one needs topology to account for the fact that two objects or events are continuously connected, or for the relation of something being inside, or surrounding something else, or being a hole.

According to [6] we distinguish three levels for the description of spatial entities: the mereological level (mereology), the topological level (topology), and the morphological level (morphology). Topology is concerned with space-relevant properties and relations as connection, coincidence, touching, and continuity, whereas morphology analyses the shape, form and size of spatial entities. We sketch here some fragments of such theories.

Mereology. [11] We collect typical axioms of the reflexive variant of the part-whole relation, denoted by $\overline{\mu}$. Some of them are formulated as basic axioms for μ in section 5.1. In the sequel we use the following definition for reflexive overlap: $ov(x,y) =_{df} \exists z(z\ \overline{\mu}\ x \wedge z\ \overline{\mu}\ y)$

1. $x\,\overline{\mu}\,x$
2. $x\,\overline{\mu}\,y \wedge y\,\overline{\mu}\,z \to x = y$
3. $x\,\overline{\mu}\,y \wedge y\,\overline{\mu}\,z \to x\,\overline{\mu}\,z$
4. $\neg x\,\overline{\mu}\,y \to \exists z(z\,\overline{\mu}\,x \wedge \neg ov(z,y))$ (supplementation axiom)
5. $\forall x(Ind(x) \to \exists z(Sit(z) \wedge \forall u(u\,\overline{\mu}\,x \to u \rhd z)$
6. $\forall xy(Ind(x) \wedge Ind(y) \to \exists z(Ind(z) \wedge x\,\overline{\mu}\,z \wedge y\,\overline{\mu}\,z))$
7. $\exists x\phi(x) \wedge \forall y(\phi(y) \to Ind(y)) \to$
   $\forall s(Sit(s) \to \exists z(Ind(z) \wedge (\forall u(ov(u,z) \longleftrightarrow \exists v(v \rhd s \wedge \phi(v) \wedge ov(u,v)))))$ [12]

Using these axioms one may introduce the mereological sum $x \sqcup y$, the mereological intersection $x \sqcap y$, and the mereological difference $x - y$ between two individuals. The ternary part-whole relation $\nu(x,y,z)$ is domain specific and depends on the universal z. We demonstrate the use of the relation $\nu(x,y,z)$ by an example. Let U_T be a universal whose instances are trees (as plants); then $\nu(x,y,U_T)$ describes the part-whole relation related to a certain granularity of trees. A biologist is not interested to describe the structure of the atoms as parts of trees. The description of the the part-whole relation related to the macroscopic biological structure of trees has to introduce a formal system of axioms $Ax_\nu(U_T;U_1,\ldots U_k)$ of the following kind. The universals $U_1,\ldots,U_k$ describe entities being admissible parts of the instances of U_T. Then the following axioms capture the intention of "being a part in the sense of U_T".

1. $\forall xy(\nu(x,y,U_T) \to \exists z(z :: U_T\ \wedge\ x\,\overline{\mu}\,z\ \wedge y\,\overline{\mu}\,z\ \wedge\ x\,\overline{\mu}\,y))$
2. $\forall xy(\nu(x,y,U_T) \to (\bigvee_{i\leq k} x :: U_i \vee x :: U)\ \wedge\ (\bigvee_{i\leq k} y :: U_i \vee y :: U))$
3. $\forall x(\bigvee_{i\leq k} x :: U_i \to \exists z(z :: U_T\ \wedge\ x\,\overline{\mu}\,z))$.

Among the universals $U_1,\ldots,U_k$ are surely such describing the notions "branch of a tree", "leaf of a tree", "trunk of a tree".

[11] We distinguish beetween ontological basic axioms and domain-specific axioms. It is an open qestion whether any axiom concerning an ontologically basic relation should be considered as basic. This might be true for the relation μ but surely not for the ternary relation $\nu(x,y,z)$ since the universal z may be associated to a specific domain transmitting properties to $\nu(x,y,z)$.

[12] This axiom is an adaptation of the comprehension schema of ZF to the area of urelements; the analogs of sets are situations.

Topology. There are several possibilities to formalize topological phenomena; here, we will take into consideration the relation of coincidence, based on the ideas of Brentano [9] and the notion of a boundary. The relation of coincidence pertains to boundaries. Let $\bowtie$ denote the relation of concidence and $b(x, y)$ let mean "x is boundary of y". Let $bd(x) =_{df} \exists y(b(x, y)$ and $Ip(x, y) =_{df} x \,\overline{\mu}\, y \wedge \forall z(b(z, y) \rightarrow \neg ov(x, z))$ (interior parthood). The following axioms are discussed in [41], [44].

1. $x \bowtie x$
2. $x \bowtie y \rightarrow y \bowtie x$
3. $x \bowtie y \wedge y \bowtie z \rightarrow x \bowtie z$
4. $b(x, y) \rightarrow Top(y)$
5. $b(x, y) \rightarrow x \,\overline{\mu}\, y$
6. $b(x, y) \wedge b(y, z) \rightarrow b(x, z)$
7. $bd(x) \rightarrow \exists y \exists z(b(x, y) \wedge Ip(z, y))$

Boundaries can be understood as surfaces of topoids; surfaces can have boundaries which we call lines, and lines can have boundaries called points. We introduce further relations $b_l(x, y)$ ("x is a boundary of the surface y"), $b_p(x, y)$ (" x is a boundary of the line y"). We introduce the definitions $bd_l(x) =_{df} \exists y b_l(x, y)$, and $bd_p(x, y) =_{df} \exists y b_p(x, y)$. Then we assume following axioms:

1. $b_l(x, y) \rightarrow bd(y)$
2. $b_p(x, y) \rightarrow bd_l(y)$
3. $b_l(x, y) \rightarrow x \,\overline{\mu}\, y$
4. $b_p(x, y) \rightarrow x \,\overline{\mu}\, y$
5. $x \bowtie y \rightarrow (bd(x) \wedge bd(y)) \vee (bd_l(x) \wedge bd_l(y)) \vee (bd_p(x) \wedge bd_p(y))$

The (strong) connectedness of a topoid may be defined as follows. $C(x) =_{df} \forall yz(x = y \sqcup z \wedge Top(y) \wedge Top(z) \rightarrow \exists rs(r \,\overline{\mu}\, y \wedge b(r, y) \wedge b(s, z) \wedge s \,\overline{\mu}\, z \wedge r \bowtie s))$. Analogously, the connectedness of boundaries is defined by $C_s(x) =_{df} \forall yz(x = y \sqcup z \wedge bd(y) \wedge bd(z) \rightarrow \exists rs(r \,\overline{\mu}\, y \wedge s \,\overline{\mu}\, z \wedge r \bowtie s))$. $C_l(x)$ denotes the connectedness predicate for lines (as boundaries of surfaces). Using the relations μ, $b(x, y)$, $b_l(x, y)$, $b_p(x, y)$ and $\bowtie$ a mereotopological theory of pure topoids (topoids separated from other categories of individuals) could be developed.

If we want to integrate other categories of individuals, say substances, in this theory then certain complications arise, and it seems that a full developed mereo-topological theory including all kinds of individuals does not yet exist. We discuss some of these problems for the case of substances. An individual substance in our sense can be understood as an amount of "physical matter".[13] The world of pure topoids and the world of substances are connected by a basic

[13] To be more precise: an individual substance is the carrier of a physical matter, because a physical matter has already certain qualities (moments) as gravity, density, mass, and energy.

relation $occ(x, y)$ having the intuitive meaning "the individual x occupies the topoid y".[14]

The following axiom seems to be plausible:

$$\forall x(Subst(x) \rightarrow \exists t(Top(t) \wedge occ(x, t)).$$

Substances may have boundaries, but the boundaries of substances are not the same as the boundaries of their occupied topoid. Thus, we introduce a new boundary-relation $bs(x, y)$ having the meaning "x is a substantial boundary of the substance y". Then, the following axiom could be acceptable;

$$\forall xtyz(Subst(x) \wedge occ(x, t) \wedge bs(y, x) \wedge b(z, t) \rightarrow z \bowtie y)$$

The connectedness of a substance can be defined as follows. $Cn(x) =_{df} Subst(x) \wedge \forall t(occ(x, t) \rightarrow C(t))$, i.e. every topoid occupied by x is connected. This definition can be extended to the connectedness of a boundary. In contradistinction to [39] we admit substances without connected boundaries. The following axiom says that every connected substance has a substantial boundary being itself connected.

$$\forall x(Subst(x) \wedge Cn(x) \rightarrow \exists y(bs(y, x) \wedge Cn(y)).$$

Morphology. To describe the form of an object in [6] we adopt a relation of *congruence*, denoted by $\sim$, between regions whose intended meaning is: "two regions are congruent if they have the same shape and size". For every topoid t one could introduce a universal U_t whose instances are topoids that are congruent with t. This would lead to a theory of shapes for pure topoids, separated from a theory substances. The universals U_t are sub-universals of more general ones; we are interested in particular in the universal U_{sp} whose instances are spheres and U_{cb} whose instances are cubes. A minimal sub-universal of U_{sp} has all spheres with the same radius as its instances.

Again, there is the problem how to connect the morphological theory of pure topoids to the world of substances. Our approach is the following. There is a universal U_{sh} whose instances are shapes; shapes are understood as moments, i.e. we have the axiom $\forall x(x :: U_{sh} \rightarrow Mom(x))$. Obviously, if a substance has a shape then it has a boundary. Hence, we admit the following axiom

$$\forall xy(Subst(x) \wedge y :: U_{sh} \wedge y \succ: x \rightarrow \exists z(bs(z, x))$$

Shapes are transmitted from substances to the topoids they occupy; we say that a topoid t has a shape related to s if there is substance whith shape s occupying t. Assume there is a substance s with shape a, and s moves from a

[14] It seems to be that the relation *occ(x,y)* is assumed to draw *fiat boundaries.* A fiat boundary (excluding cognitive decision) through a substance s could be defined as a suitable mereological partition of s which is derived from a prior partition of the pure topoid occupied by s.

position p_1 to a position p_2, and assume that a is invariant during this movement then the topoids occupied by s in positions p_1 and p_2 are congruent.[15]

6.3 Medicine

In the sequel we use mereotopological notions to give some hints as to how the macroscopic anatomy of a kidney could be described. We begin with a specification of the used domain specific universals and relations. The signature contains the following universals among others: $U(\text{organ}), U(\textit{human-being}), U(\textit{kidney})$, $U(colour), U(brown - reddish), U(shape)$.

Every kidney is an organ.
$\forall x(x :: U(kidney) \to x :: U(organ))$

Every human being has two kidneys.[16]
$\forall x(x :: U(human - being) \to \exists uv(u \neq v \wedge u :: U(kidney) \wedge v :: U(kidney) \wedge u\,\mu\,x \wedge v\,\mu\,x)$.

Every kidney is a connected substance.
$\forall x(x :: U(kidney) \to Cn(x))$

We describe some notions which are important for the modelling of the anatomic structure of human organs in general. An important property of a boundary is its *concavity* or *convexity*. To define concavity and convexity one may use the theory of pure topoids. For every connected topoid t there is a uniquely determined minimal connected topoid s such that $t\,\overline{\mu}\,s$ and s is convex. s is called the convex closure of t, denoted by $convcl(t) = s$. A connected topoid t is convex if every cut through t by a plane p implies that $(p \sqcap t) - bd(t)$ is a part of the interior of t. A part s of the boundary of a connected topoid t is said to be convex if it coincides with a part of the convex closure $convcl(t)$ of t. Obviously, for every convex part s of the boundary of t there is a maximal convex part s_1 of $bd(t)$ which is connected and contains s as a part. The concave part of $bd(t)$ is the complement of the mereological sum of all maximal convex parts of $bd(t)$. The boundary of a connected topoid is the mereological sum of its convex parts and its concave parts.

One problem is, how to define a cut through a connected topoid by a plane. Here, one has to use some suitable universals whose instances are topoids with a certain shape; possible candidates are the universals U_{cb} and U_{sp} describing cubes resp. spheres. One may, for example, select a suitable sphere s and a suitable cube c such that $s_1 = s - c$ and $s_2 = s \sqcap c$ are congruent and $b(s_1), b(s_2)$

[15] To make this precise one has to understand what it means that a moment is invariant during a time period.

[16] Many of the axioms in medicine are defaults, i.e they describe the normal case. The exceptions may be treated by using non-monotonic inference systems. GOL provides the subsystem *KR1-Prolog* for the formalization of non-monotonicity; systems of this kind were proved in [22] to be very useful in medical applications.

have maximal parts which coincide. These parts can be used as a substitute for a plane, i.e. we may consider intersection of s_1 with arbitrary connected topoids.

The boundary of a kidney is the mereological sum of its convex boundary and its concave boundary. The convex boundary and concave boundary of a kidney are themselves connected. The convex boundary and concave boundary of a kidney have coincident line boundaries. The convex boundary of a kidney is the mereological sum of its *facies anterior* and its *facies posterior*. The *margo lateralis* and the *margo medialis* can be defined by a suitable cut through the kidney; this cut determines a line-boundary on the convex boundary and a line-boundary on the concave boundary.

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References

1. Alferes,J, H. Herre, L.M.Pereira: Partial Models of Extended Generalized Logic Programs;in: Computational Logic -CL2000, J. Lloyd et a. (eds.), LNAI vol. 1861, 149-163
2. Aristoteles. Philosophische Schriften, Felix-Meiner Verlag, Hamburg, 1995
3. Baader,F. A formal definition for the expressive power of terminological knowledge representation languages; *Journal of Logic and Computation*, 6:33-54, 1996
4. Barwise, J., J. Perry. Situations and Attitudes; Bradford Books, The MIT Press, 1983
5. Barwise, J. Situations in Logic. CSLI No. 17, 1989
6. Borgo,S.,N. Guarino, and C. Masolo. Qualitative Spatial Modelling based on Parthood, Strong Connection and Congruence; LADSEB-CNR, Rep. 03/96 (1996)
7. Borgo,S.,N. Guarino, and C. Masolo. Towards an Ontological Theory of Physical Objects.
8. Brachman,R., J. Schmolze. An overview of the KL-ONE knowledge representation system. *Cognitive Science* 9 (2) : 171-216, 1985
9. Brentano, F. Philosophische Untersuchungen zu Raum Zeit und Kontinuum; Felich Meiner Verlag, Hamburg, 1976
10. Casati,R. A.C. Varzi. The Structure of Spatial Localization; *Philosophical Studies , 82* (1996), 205-239.
11. Chisholm,R.M.: Boundaries as Dependent Particulars, *Grazer Philosophische Studien*, 20, 87-96
12. Chisholm,R.M: On Metaphysics, Minneapolis: University of Minnesota Press, 1989
13. Cocchiarella, N.B. Formal Ontology. In Burkhardt,H. und B. Smith (eds.) *Handbook of Metaphysics and Ontology*, Philosophia Verlag, München, pp. 640-647, 1991
14. Copeland, J.B. CYC: A case Study in Ontological Engineering; 1997, http://www.phil.indiana.edu/ejap/copeland976.2.html

15. http://www.cyc.com
16. Doyle,J., R.S.Patil. Two theses of knowledge representation: language restriction, taxonomic classification, and the utility of representation services; *Artificial Intelligence* 48 (1991), 261-297
17. Engelfriet,J., H. Herre: Stable Generated Models, Partial Temporal Logic and Disjunctive Defaults; *Journal of Logic Pprogramming*, 41 (1999), 1-25
18. Fensel, D. Ontologies: Silver Bullet for Knowledge Management and Electronic Commerce.
19. Galen-System. http://www.opengalen.org
20. Genesereth, M.R., R. E. Fikes: Knowledge Interchange Format, Version 3.0, Reference Manual, Logic Group Report Logic-92-1, Computer Science Department, Stanford University, Stanford, California
21. Gelfond,M., Lifschitz,V.: The stalbe Model Semantics for Logic Programs; in: R.A.Kowalski, K.A.Bowen (eds.) *Proc. of the ILPS'88*, MIT Press, Cambridge, MA, 1988, 1070-1080
22. Gartner,J., T. Swift,AS. Tien, C.V. Damasio, L.M.Pereira: Psychiatric Diagnosis from the Viewpoint of Computational Logic; in: Computational Logic CL2000, J. Lloyd et a. (eds.), LNAI vol. 1861, 1362-1376
23. Genesereth,M.R. and Nilsson N.J. Logical Foundation of Artificial Intelligence, Morgan Kaufmann, Los Altos, California, 1987
24. Gruber, T.R. A translation approach to portable ontology specifications. In *Knowledge Acquisition* vol. 5, pp 199-220, 1993
25. Gruber,T.R.: Towards Principles for the Design of Ontologies Used for for Knowledge Sharing, *International Journal of Human and Computer Studies*, vol. 43, 907-928 (1995)
26. Guarino,N. Ontologies and Knowledge Bases: Towards a Terminological Clarification; in N.J.L. Mars (ed.) *Towards Very Large Knowledge Bases*, IOS Press (1995), Amsterdam
27. Guarino,N., P. Giaretta: Ontologies and Knowledge Bases (Towards a Terminological Clarification) in N.J.I. Mars (ed.) *Towards Very Large Knowledge Bases* , IOS Press, 1995, Amsterdam
28. Guarino,N. Formal Ontology and Information Systems. in N Guarino (ed.) *Formal Ontology in Infomrmation Systems* Proc. of FOIS'98, Trento, Italy, Amsterdam, IOS-Press
29. Guarino, N. Some Ontological Principles for Designing Upper Level Lexical Resources, First Conference on Language Resources and Evaluation, May 1998
30. Guarino, N., Poli,R. in *Formal Ontology in Conceptual Analysis and Knowledge Representation*
31. Hao Wang: Reflections on Kurt Gödel Bradford Book, MIT Press, 1987
32. Herre,H., Wagner,G.: Stable Models are Generated by a Stable Chain: *Journal of Logic Programming* 30 (2), (1997), 165-177
33. Herre,H., Jaspers, J., Wagner,G.: Partial Logics with Two Kinds of Negation of a Foundation for Knowledge-Based Reasoning; in: D.M.Gabbay,H. Wansing (eds.), *What is Negation?*, 121-159, Kluwer Academic Publishers, 1999
34. Kiefer,M., G. Lausen, J. Wu. Logic Foundation of Object-Oriented and Frame-Based Languages; TR 93-06; Department of Computer Science, SUNY at Stony Brook, Stony Brook, NY 11794
35. Levesque,H.J., R.J.Brachman. Expressiveness and tractability in knowledge representation and reasoning. *Comp. Intell.*, 3 , 78-93

36. Mark, W. Ontology as Knowledge Base Architecture, in Proc. of the *Banff Knowledge Acquisition Workshop, 1995*
37. Mulligan,K.,P. Simons, B. Smith: Truth-Makers. *Philosophy and Phenomenological Research* 44,(1984), 287-321
38. Musen,M.: Dimensions of Knowledge Sharing and Reuse. In *Computer and Biomedical Research*, vol. 25, p.435-467, 1992
39. Smith, B. On Substances, Accidents and Universals. In defence of a constituent Ontology. *Philosophical Papers* 26, (1997), 105-127
40. Smith,B. Logic and the Sachverhalt. *The Monist*, 72 : 1 (1989), 52-69
41. Smith,B. Boundaries: An Essay in Mereotopology, from L. Hahn,ed. *The Philosophy of R. Chisholm*, LaSalle: OpenCourt, 1997, 534-561
42. Smith,B. Truthmaker Realism, *Australasian Journal of Philosophy*, Sept. 1999
43. Smith, B., B. Brogaard. Sixteen Days: The Ontology of Fetal Development, Sept. 1999, Draft XXV
44. Smith,B., A.C. Varzi. Fiat and Bona Fide Boundaries; forthcoming in *Philosophy and Phenomenological Research*
45. Tarski, A.: The Semantics Conception of Truth and the Foundation of Semantics, *Philosophy and Phenomenological Research*, vol. 4 (1944), 341-375
46. Tarski, A. What is elementary Geometry? in: The axiomatic Method (eds. L. Henkin, P. Suppes , A. TArski), North-Holland, 1959, p.16-29
47. Varzi, A.C. Parts, Wholes, and Part-Whole Relations: The Prospects of Mereotopology; *Datas and Knowleldge Engineering 20*, (1996), 259-286.
48. Varzi,A.C. Boundaries, Continuity, and Contact, *Nous, 31*, (1997), 26-58
49. Woodger,J.H. Studies in the Foundation of Genetics in: The axiomatic Method (eds. L. Henkin, P. Suppes , A. Tarski), North-Holland, 1959, p. 408-428

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