Template Matching on Vector Fields using Clifford Algebra

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Due to the amount of flow simulation and measurement data, automatic detection, classification and visualization of features is necessary for a thorough inspection. Therefore, many automated feature detection methods have been developed in the last years. However, one feature class is visualized afterwards in most cases, and many algorithms have problems in the presence of noise or superposition effects.
In contrast, image processing and computer vision have robust methods for feature extraction and the computation of derivatives of scalar fields. Furthermore, interpolation and other filter can be analyzed in detail. An application of these methods to vector fields would provide a solid theoretical basis for feature extraction. We suggest Clifford algebra as a mathematical framework for this task.
Clifford algebra provides a unified notation for scalars and vectors as well as a multiplication of all the basis elements. For the d-dimensional Euclidean vector space E^d with the basis $\{\mathbf{e}_1, .., \mathbf{e}_d\}$, the Clifford Algebra G_d is defined by the basis $\mathbf{e}_A$, $A \subset \{1, .., d\}, e_\emptyset = 1$, and an associative, bilinear multiplication defined by the equations $1\mathbf{e}_j = \mathbf{e}_j$, $\mathbf{e}_j\mathbf{e}_j = 1$, $\mathbf{e}_j\mathbf{e}_k = -\mathbf{e}_k\mathbf{e}_j$, $j, k = 1, .., d$, $j \neq k$. Note that multiplication is not commutative in general. The usual Cartesian vectors $\mathbf{x} = (x_1, .., x_d)^T \in \mathbb{R}^d$ are identified with $x_1\mathbf{e}_1 + ... + x_d\mathbf{e}_d$ in G_d. Arbitrary elements of the Clifford algebra are called multivectors.
Within Clifford Algebra, convolution is easily defined as

$$(\mathbf{H} *_l \mathbf{F})(\mathbf{x}) = \int_{E^d} \mathbf{H}(\mathbf{x'})\mathbf{F}(\mathbf{x} - \mathbf{x'})|d\mathbf{x'}| \quad (1)$$

where l denotes the application of the filter or template from the left. This convolution has attractive geometric properties that allow template matching on vector fields [1].
In image processing, convolution and Fourier transform are closely related by the convolution theorem. For frequency analysis of vector fields and the behavior of vector-valued filters, a Clifford Fourier transform has been derived for $k = 2, 3$ by replacing the complex i in the definition of the Fourier transform with $i_2 = e_1e_2$ and $i_3 = e_1e_2e_3$ respectively.

$$\mathcal{F}\{\mathbf{F}\}(\mathbf{u}) = \int_{E^d} \mathbf{F}(\mathbf{x})e^{(-2\pi \boldsymbol{i}_d \langle \mathbf{x}, \mathbf{u} \rangle)}|d\mathbf{x}|, d = 2, 3$$

In 3D, the Clifford Fourier kernel is commutative with every element of G_3. In 2D, it is anti-commutative with the vector elements and commutative with the rest. Convolution and other theorems have been proven, and fast algorithms for the computation of the Clifford Fourier transform exist. Therefore the computation of the Clifford convolution can be accelerated by computing it in Clifford Fourier domain.

Rotation invariant template matching based on Clifford convolution has been applied successfully for detection and thorough analysis of features in a variety of data sets [1,3]. The algorithm has also been extended to automatically compute feature based segmentations of flow data sets as the template matching provides a unified algorithm for the detection of many different features. Visualizations of the results display important structures of the flow and highlight the interesting features (see Figure 2).

Here, the authors give an overview of their work. The mathematical basis as well as the results of applying template matching to several complex flow data sets are presented and discussed.

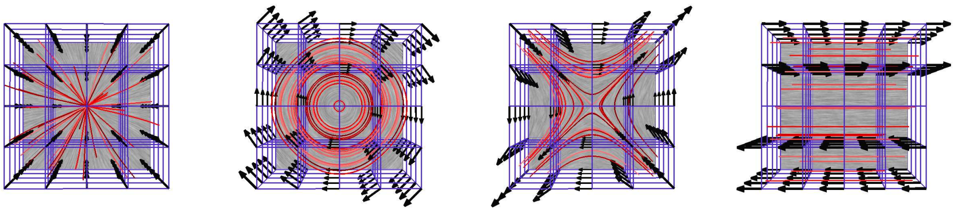

Figure 1: Some basic 3D vector-valued templates.

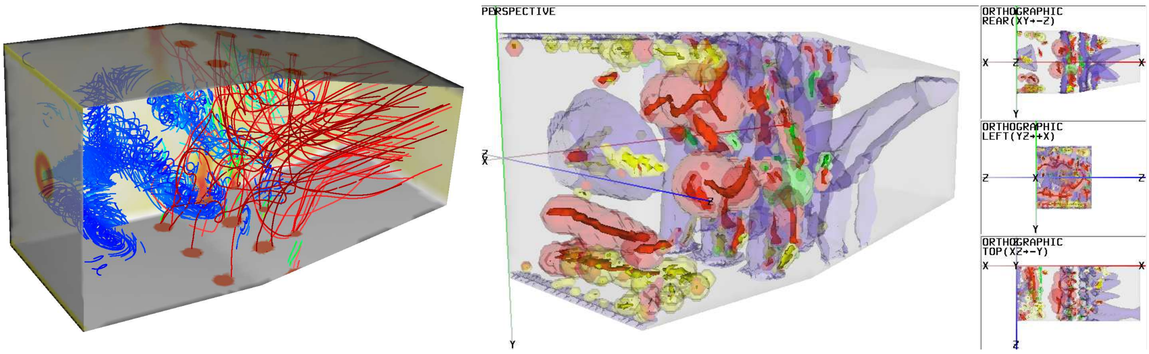

Figure 2: A gas furnace chamber. **Left**: Color coding of the velocity of the boundary to show the inflow areas. Streamlines (red) started at the top and bottom gas inflow. The gas leaves the chamber in the rear. Streamlines (blue) seeded by matching results of a 5x5x5 rotational template (threshold 0.5) display large vortices. **Right**: Segmentation of the normalized gas furnace chamber. Isosurfaces of the results (value 0.5): Red: rotations, yellow: shear flow, green: saddles. The cores of the regions are also displayed. The velocity of the original data set is displayed at an isovalue of 15 in blue.

References

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